Equivalent Expressions with the Distributive Property

Short description: Learn how the distributive property can be used to model and create equivalent expressions in this animated Math Shorts video.

Long description: This animated Math Shorts video explains how the distributive property can help students model and create equivalent expressions. In the accompanying classroom activity, students play a quick game where they identify common factors within an expression and work on a series of problems that stretch their understanding of how to apply the distributive property. While the problems begin with whole number expressions, they soon work towards algebraic notation, and eventually develop the idea that $ax + bx$ can be rewritten as $x(a + b)$.

Activity Text

Learning Outcomes
Students will be able to
- identify common factors within an expression
- rewrite an expression by applying the distributive property

Common Core State Standards: 6.EE.A.3, 6.NS.B.4

Vocabulary: Common factors, distributive property, equivalent expressions

Materials: Small counters (dried beans, pennies, etc.)

Procedure

1. Introduction (5 minutes, whole group)
Begin with a brief game to stretch students' mental math skills: Can they find the *common factors* (if any) between two numbers? Use the following list of pairs to get started.

- 5, 15 (common factor: 5)
- 8, 10 (common factor: 2)
- 2, 7 (no common factors)
- 18, 12 (common factors: 2, 3, 6)
- 50, 30 (common factors: 2, 5, 10)
- x, $5x$ (common factor: x)
- $3x$, $9x$ (common factors: x, 3)
- $7x$, 18 (no common factors)

Tie the game into the day's learning goals. Tell students that finding common factors is an important part of applying the *distributive property*. Then introduce the video, and tell students that they will learn how the distributive property can help them create *equivalent expressions*.

2. Watch the Video (10 minutes, whole group)
Show students the video. After watching, ask students why the idea of common factors is important in connecting the following three expressions:

- 5 + 15
- 5 * 1 + 5 * 3
- 5(1 + 3)

Now ask students how to create an expression equivalent to 6 + 27 using the distributive property. Set up an array of 6 and 27 using small counters, and then show that the array can be reorganized as 3 * 2 and 3 * 9, which creates similar-sized groups. From there, 3(2 + 9) is a logical step.

3. Activity (10 minutes, small groups)
In pairs, have students work through the problems on the worksheet. Each pair should have about 40 counters to use with the problems.

4. Conclusion (5 minutes, whole group)
Review the answers to the worksheet problems together. Then assess students' understanding by having them do the following:

- Find two different ways to represent the expression 10 + 20 using the distributive property.
- Figure out how to use a model or array to rewrite the expression 21 + 7 + 14 using the distributive property.
- Use a model/array to explain how 21 + 12 is the same as 3(7 + 4).

Exploring Equivalent Expressions: Problems

1. Each expression below is shown as an array. Use the distributive property to rewrite each expression in *a*(*b* + *c*) form.

4 + 16 Rewritten:	o o o o o o o o o o o o o o o o o o o o
12 + 15 Rewritten:	o o o o o o o o o o o o o o o o o o o o o o o o o o o
4x + 20 Rewritten:	x o o o o o x o o o o o x o o o o o x o o o o o
12x + 6 Rewritten:	x x o x x o x x o x x o x x o x x o

2. Build arrays for the following expressions, and then represent them using the distributive property.

3 + 12 Rewritten:	
28 + 8 Rewritten:	
$2x + 22$ Rewritten:	
$18x + 3$ Rewritten:	

3. Look at one of the problems above. How does finding common factors help you figure out how to structure the array?

4. Shawn and Sheila are having a disagreement about the expression $10x + 30x$. Shawn says that the expression is equivalent to $40x$. Sheila says that you can't add the terms together because you don't know how much x is. How can Shawn use the distributive property to show that $10x + 30x$ is equivalent to $40x$?

Exploring Equivalent Expressions: Solutions

1. Each expression below is shown as an array. Use the distributive property to rewrite each expression in *a*(*b* + *c*) form.

4 + 16 Rewritten: 4(1 + 4)	o o o o o o o o o o o o o o o o o o o o
12 + 15 Rewritten: 3(4 + 5)	o o o o o o o o o o o o o o o o o o o o o o o o o o o
4x + 20 Rewritten: 4(x + 5)	x o o o o o x o o o o o x o o o o o x o o o o o
12x + 6 Rewritten: 6(2x + 1)	x x o x x o x x o x x o x x o x x o

2. Build arrays for the following expressions, and then represent them using the distributive property.

3 + 12 Rewritten: 3(1 + 4)	array: 3 across, 1 and 4 down
28 + 8 Rewritten: 4(7 + 2)	array: 4 across, 7 and 2 down
$2x + 22$ Rewritten: $2(x + 11)$	array: 2 across, one x and 11 units down
$18x + 3$ Rewritten: $3(6x + 1)$	array: 3 across, six x and 1 unit down

3. Look at one of the problems above. How does finding common factors help you figure out how to structure the array?

Answer: You can use a common factor to determine the width of an array. So if 3 is a common factor, then the array will be 3 units across. Then you can structure the rest of the array according to that width.

4. Shawn and Sheila are having a disagreement about the expression $10x + 30x$. Shawn says that the expression is equivalent to $40x$. Sheila says that you can't add the terms together because you don't know how much x is. How can Shawn use the distributive property to show that $10x + 30x$ is equivalent to 40x?

Answer: $10x + 30x$ is the same as 40x, because x is a common factor. So $10x + 30x = x(10 + 30)$. And since $10 + 30 = 40$, $10x + 30x = 40x$.